78-512

Cutout Reinforcement of Stiffened Cylindrical Shells

J.A. Cervantes* and A.N. Palazotto†
Wright-Patterson Air Force Base, Dayton, Ohio

A study was carried out to determine the optimum placement and volume of a reinforcing frame around a cutout in an axially loaded stringer and ring and stringer stiffened cylindrical shell. The problem was analyzed using the linear bifurcation portion of STAGS (Structural Analysis of General Shells). Four parameters were varied; stringers vs rings and stringers, cutout size, ratio of frame volume to cutout volume, and frame position. It appeared that the frame's position next to the cutout edge was the most effective, which is simply a confirmation of a well-known fact concerning reinforcement of shell cutouts. However, there was a relative maximum in the frame distance vs critical load curves for a frame positioned away from the cutout edge at a low ratio of frame to cutout volume.

Nomenclature

A_{sf}, b_{sf}, h_{sf}	= area, width, and depth of reinforcing frame
a	= hole radius or half-width of space cutout
d	= distance of frame from the centerline of the cutout
E	= modulus of elasticity
G	= shear modulus
h	= shell thickness
$I_{x \text{ or } y}, I_z$	= moment of inertia of reinforcing frame
L	= length of the shell
P_0	= buckling load of shell without a cutout and without a reinforcing frame
P_1	= buckling load of shell with a cutout but without a reinforcing frame
P_{cr}	= buckling load
R	= radius of cylinder
u, v, w	= displacements in the axial, circumferential, and radial directions, respectively
W_{sf}/W_0	= ratio of the reinforcing frame volume to the volume of the removed cutout material
WOA	= volume per unit area of the removed cutout material
x, θ, z	= the coordinates in the axial, circumferential and radial directions, respectively
ν	= Poisson's ratio

Introduction

THE cylindrical shell is an important aerospace structural element. The semimonocoque airplane shell structure of the 1930's was typified by a very thin skin supported by much stiffer longitudinal and transverse reinforcing members. As a result, buckling of the thin skin panels was primarily a local phenomenon. With the advent of missiles and the increase in flight speed, aircraft skins have become thicker, and skins and reinforcement are of similar rigidity. Buckling changed from a local characteristic to a problem of general shell instability. A further complication is encoutered when an access door or similar hole must be inserted in the structure, thus weakening an otherwise very efficient structure. The purpose of this paper is to determine the strengthening effects of a reinforcement around a rectangular cutout in a stiffened cylindrical shell.

Submitted March 2, 1978; presented as Paper 78-512 at AIAA/ASME 19th Structures, Structural Dynamics, and Materials Conference, Bethesda Md., April 3-5, 1978; revision received July 12, 1978.

Index categories: Structural Design; Structural Stability.

*Aerospace Engineer, Aeronautical Systems Division.

†Professor, Astronautics and Aeronautics Dept., AFIT School of Engineering. Member AIAA.

Previous research into the presence of cutouts in cylindrical shell structures can be traced to work performed by Starnes,[1] who investigated the buckling of a thin unstiffened cylindrical shell with a single cutout, both experimentally and theoretically. He conducted two series of experiments, one on shells made of Dupont's Mylar with a lap seam; and the other series of tests on seamless electroformed copper shells. The parameters ranged between $400 < R/h < 960$ and $0 < a/R < 0.5$; where a was the hole radius. The experimental buckling loads, displacements, and the stress distribution applied at the end of the shell were correlated with a theoretical parametric study performed by means of a Rayleigh-Ritz-type approximation. The results of these experiments led to the conclusion that the governing parameter of the problem was related to a^2/Rh. It has been shown in Ref. 1 that the parameter

$$mu = \tfrac{1}{2}\,[12(1-\nu^2)]^{1/4}\,(a^2/Rh)^{1/2} \tag{1}$$

governs the prebuckling stress distribution and displacements for a circular cylinder with a circular hole in its side. It should be noted that the effect of the hole is very localized in nature. Both membrane and bending stress increments occur; but the bending stress increment is always much less than the membrane stress increment. It should be appreciated that the maximum membrane stress will rise significantly above the stress value obtained for a hole in a flat plate. Work related to stiffened cylindrical shells without cutouts is discussed by Singer[2] and will not be presented herein.

Palazotto[3] explored the validity of using linear bifurcation theory for the buckling analysis of stringer and ring stringer stiffened cylindrical shells with rectangular cutouts employing clamped boundary conditions. The analysis was performed using the STAGS (Structural Analysis of General Shells)[4] computer program. This program incorporates linear bifurcation theory, nonlinear collapse analysis, discrete stiffening, and smeared stiffening theory.

The buckling loads of stringer and ring and stringer cylindrical shells were computed using both the linear bifurcation analysis and the nonlinear collapse analysis capability of STAGS. When the results for the linear and nonlinear analyses compared within a few percent, Palazotto concluded that linear bifurcation theory adequately predicted the buckling load of a stiffened cylindrical shell with cutouts. The importance of this fact is that a nonlinear collapse analysis is much more expensive than a linear bifurcation analysis in terms of computer time.

Model Parameters

Model Description

In this paper, a stiffened circular cylindrical shell with a reinforcing frame around the cutout was analyzed for a

Table 1 Model parameters which were varied

Internal stiffening	Total cutout size: $2a$		
a. Stringers only	a. 12×12 in.		
b. Rings and stringers	b. 24×24 in.		
Ratio of reinforcing frame volume to removed cutout volume, W_{sf}/W_0	Reinforcing frame location d, in.[a]		
	Position	$2a = 12$ in.	$2a = 24$ in.
1. 0.5	1	6.0	12.0
2. 1.0	2	7.5	13.5
3. 1.5	3	9.0	15.0
4. 2.0	4	12.0	18.0
·	6	18.0	

[a] Frame distance d from the centerline of the cutout.

variety of parameters. The basic geometry was held contant while four parameters were varied independently as indicated in Table 1. The shell geometry is shown in Fig. 1. The overall dimensions were chosen so that results could be compared with previous work by Palazotto.[3] The dimensions given approximate a large missile interstage.

The ring and stringer geometries were decided upon in order to approximate those that would be expected in actual applications. A 2:1 ratio of the height to width was chosen as a compromise between good bending stiffness and yet reasonable torsional stiffness. Spacing of each ring or stringer was set at 3 in. The two cutout sizes used were $a = 6$ in. and $a = 12$ in. corresponding to 12×12-in. and 24×24-in. cutouts, respectively. The value of $a = 12$ in. approximated the largest cutout size for which linear theory has been validated.[3]

The reinforcing frame volume to removed cutout volume ratios correspond to the situation where the reinforcing frame weighs from one-half to twice the weight of the material removed for the cutout. This was considered as large a range as would be of practical interest.

Note that the frame positons indicated in Table 1 are different for $a = 6$ in. and $a = 12$ in. The maximum frame positions were chosen as being the maximum distance at which the frame would have a significant impact on the buckling load as determined in Ref. 3. In addition, the total frame volume remained constant as the frame position was varied. This was necessary in order to compare the most efficient frame position for a given volume ratio.

The equations for the geometric properties of the reinforcing frame have been established using the following expressions. The volume per unit area of the cutout for the stringer stiffened shell is $WOA = 0.19068$ in.^{3}/in.^{2}; and for the ring and stringer stiffened shell $WOA = 0.20975$ in.^{3}/in.^{2}. The cross-sectional area of stiffening frame is given by

$$A_{sf} = a^2 \, WOA\,(W_{sf}/W_0)/2d \tag{2}$$

where $2a$ is the total cutout dimension and d is the distance from the cutout centerline to the stiffening frame. For a square cross section, the width and height of the stiffening frame become

$$b_{sf} = h_{sf} = (A_{sf})^{1/2} \tag{3}$$

The moments of inertia equal

$$I_{x\,\mathrm{or}\,y} = I_z = \frac{b_{sf} h_{sf}^3}{12} = \frac{h_{sf}^4}{12} \tag{4}$$

The torsional stiffness constant, for $b/h = 1$ is[5]

$$J_{sf} = 0.1406 b_{sf}^3 h_{sf} \tag{5}$$

For the present value of G (aluminum)

$$GJ_{sf} = 527382.\ h_{sf}^4 \text{psi} \tag{6}$$

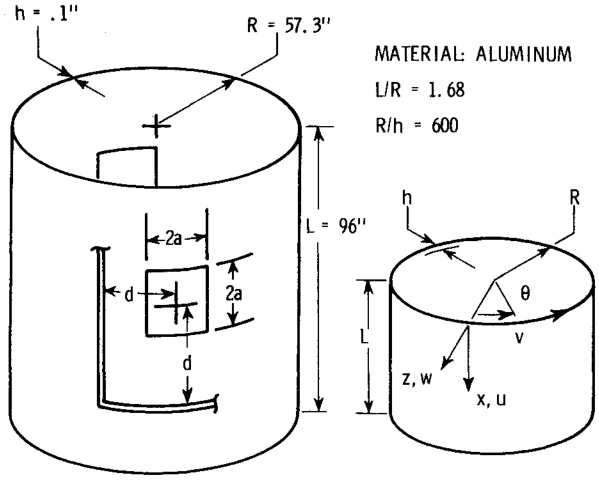

Fig. 1 General shell configuration.

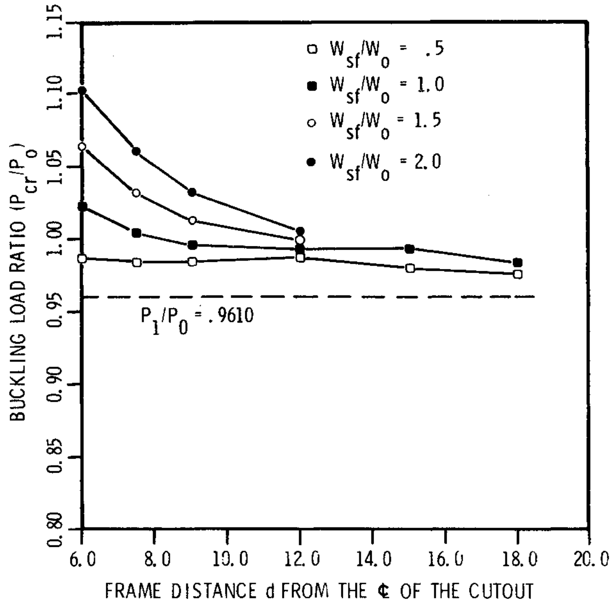

Fig. 2 P_{cr}/P_0 vs d for stringer stiffened shells with cutout size $2a = 12$ in.

Mathematical Modeling

As previously stated, the shells investigated in this paper are stiffened circular cylindrical shells with reinforcements around two symmetrically placed rectangular cutouts. The linear bifurcation capability of STAGS was used to predict buckling loads, as opposed to a nonlinear collapse analysis which would have required considerably more computer time. Palazotto[3] substantiated the use of linear bifurcation theory for the stiffened cylindrical shells considered herein.

He found that the buckling load predicted by linear bifurcation theory was, at most, only 6.87% higher than the collapse load predicted by nonlinear theory. This is an acceptable level of accuracy since the expense of nonlinear analyses would severely limit the affordable range of parameters that could be investigated.

Smeared stiffener theory was used to represent the uniformly spaced rings and stringers stiffening the entire shell. An alternate method was to specify a discrete stiffener for ring or stringer. The additional input time required for discrete stiffening was not justified by a commensurate increase in accuracy of the buckling load as shown by Palazotto.[3] He indicated that the difference in buckling loads using smeared stiffeners and discrete stiffeners amounted to less than 4% for a similar problem.

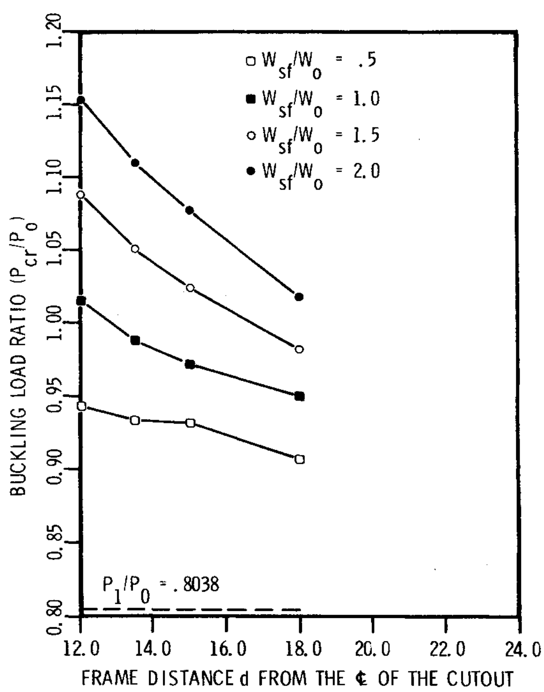

Fig. 3 P_{cr}/P_0 **vs** d **for stringer stiffened shells with cutout size** $2a=24$ **in.**

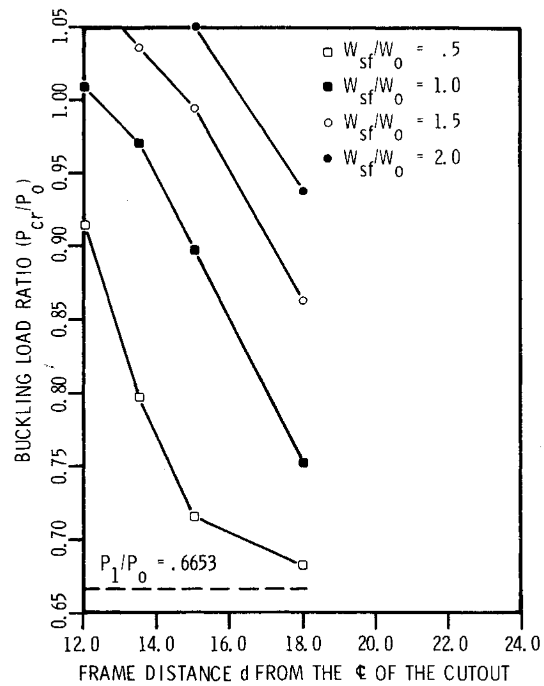

Fig. 5 P_{cr}/P_0 **vs** d **for ring and stringer stiffened shells with cutout size** $2a=24$ **in.**

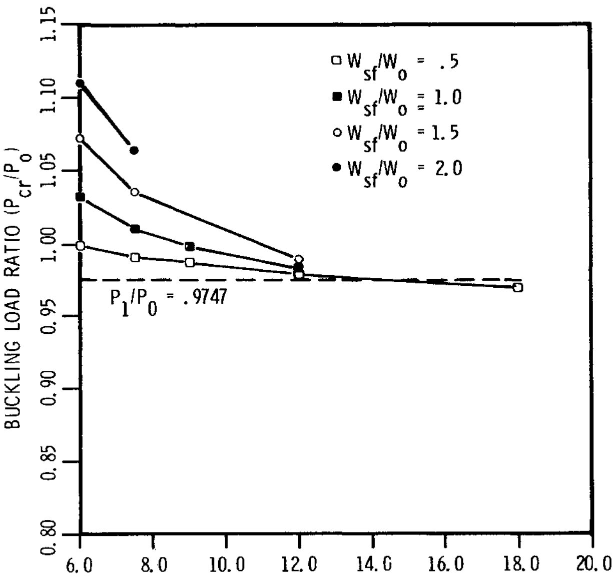

Fig. 4 P_{cr}/P_0 **vs** d **for ring and stringer stiffened shells with cutout size** $2a=12$ **in.**

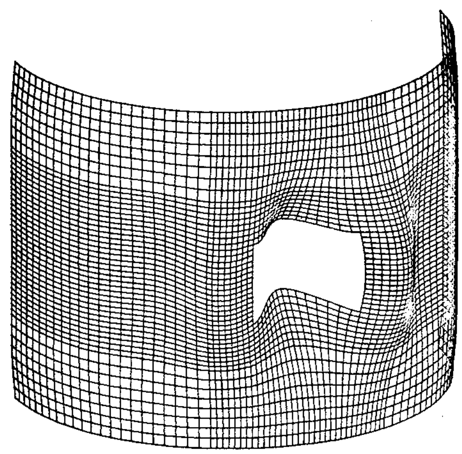

Fig. 6 Linear displacements for a stringer stiffened shell with no reinforcing frame, $W_{sf}/W_0=0$, $2a=24$ **in.**

Thus, based upon the number of mesh points incorporated into the previously cited reference, the authors used a minimum of 776 mesh points distributed over one-eighth the shell with refinement of mesh lines in the vicinity of the cutouts. The reinforcing frame around the cutout was modeled using discretely located stiffeners. Boundary conditions along the shell's curved edge included a prebuckling clamped edge with a displacement in the axial direction. A fully clamped buckling boundary was also applied.

Numerical Results

The buckling load for the particular shell configurations investigated is indicated by P_{cr}. P_0 is the buckling load of a shell with the same type of overall shell stiffening (stringers only or rings and stringers) as the shell in question, but one that does not have cutouts or reinforcing frames. P_l is the buckling load of a shell with a cutout but without reinforcing frames.

The buckling load ratio P_{cr}/P_0 is plotted vs the distance d of the frame from the centerline of the cutout in Figs. 2-5. Figures 2 and 3 are for stringer stiffened shells, and Figs. 4 and 5 are for shells stiffened by rings and stringers. Figures 2 and 4 have the cutout dimensioned as $2a=12$ in.; Figs. 3 and 5, $2a=24$ in. P_l/P_0 has been shown for reference. An inspection of Figs. 2 through 5 indicates that the most effective position for the reinforcing frame is always along the cutout edge.

It has been observed[3] that the general collapse shape in the vicinity of a cutout is similar to that of a shell's displacement field under a linear prebuckling analysis. Thus, it is possible to appreciate the ideal frame position by comparing the linear displacement fields of shells with the reinforcing frames in various locations. The linear displacements of a stringer stiffened shell with a cutout size $2a=24$ in. and volume ratio $W_{sf}/W_0=2$, are shown in Figs. 6 and 7 for the case with no frame and the case with a frame at the cutout edge, respectively. In Fig. 6, notice the large radial displacements along

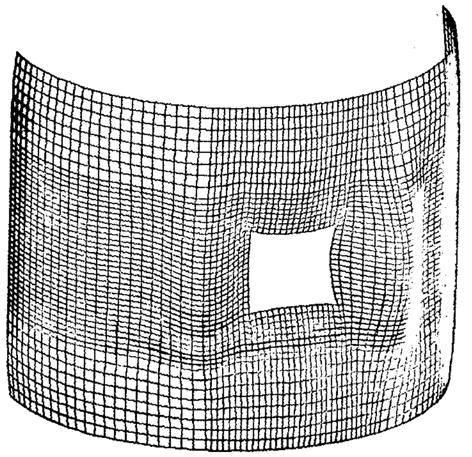

Fig. 7 Linear displacements for a stringer stiffened shell with a reinforcing frame at position 1, $W_{sf}/W_0 = 2$, $2a = 24$ in.

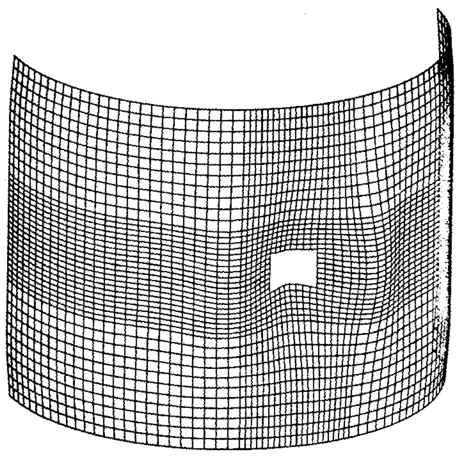

Fig. 8 Linear displacements for a stringer stiffened shell with no reinforcing frame, $W_{sf}/W_0 = 0$, $2a = 12$ in.

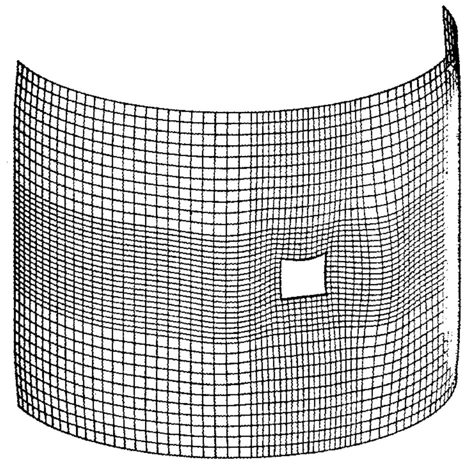

Fig. 9 Linear displacements for a stringer stiffened shell with a reinforcement at position 1, $W_{sf}/W_0 = 0.5$, $2a = 12$ in.

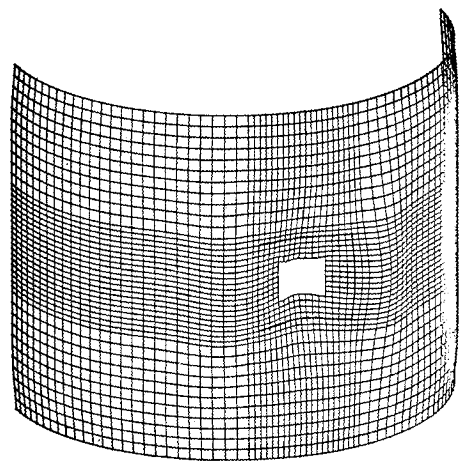

Fig. 10 Linear displacements for a stringer stiffened shell with a reinforcing frame at position 4, $W_{sf}/W_0 = 0.5$, $2a = 12$ in.

the upper and lower edges of the cutout. In Fig. 7, the very definite stiffening effect of the frame along the cutout edge is readily apparent. Both the large radial displacements along horizontal edges and the large rotations along the vertical edges of the cutout have been dramatically reduced by the reinforcement.

An additional optimum position occurs for a stringer stiffened shell with cutout size $2a = 12$ in. and volume ratio $W_{sf}/W_0 = 0.5$, in which the reinforcing frame is equally effective 6 in. away from the cutout as it is along the cutout edge. This is in spite of the fact that at the outer position the cross-sectional area of the reinforcing frame is only half the area of inner frame (frame volume is independent of position), and the moments of inertia have a ratio of 1:4. The linear displacement fields are shown in Figs. 8-10 for no frame, a frame along the cutout edge, and a frame 6 in. from the cutout edge, respectively. Inspection of Figs. 8 and 9 reveals that the displacements and rotations along the cutout edge are again somewhat smaller for the frame adjacent to the cutout edge than those for the unreinforced shell. For the frame 6 in. away from the cutout, however, the displacements are not reduced as much. It is also evident, from computer output, that a reinforcing frame adjacent to the cutout significantly reduces the stresses in the vicinity of a cutout below those for an unreinforced shell. This stress reduction lessens the chance for local buckling, consequently leading to a higher shell buckling load. However, a frame at position 4 also yields a slight reduction of stress in the area of the cutout.

The trends of the curves in Figs. 2-5 indicate that a reinforcing frame positioned far removed from the cutout may

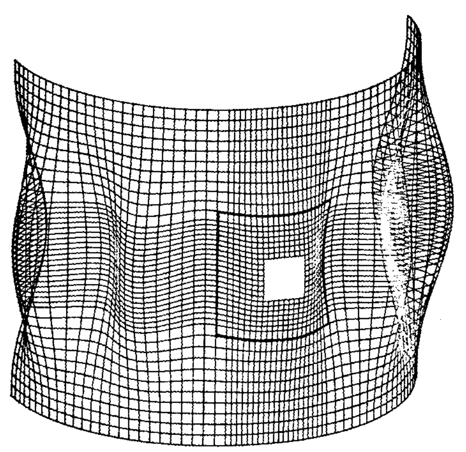

Fig. 11 Normalized buckling displacements for a stringer stiffened shell with a reinforcing frame at position 6, $W_{sf}/W_0 = 0.5$, $2a = 12$ in.

cause the shell to buckle at a load lower than the buckling load for a shell with an unreinforced cutout. This, in fact, occurs in Fig. 4 for a ring and stringer stiffened shell for a volume ratio $W_{sf}/W_0 = 0.5$, and a frame at position 6.

In order to suggest an explanation for this occurrence, consider a stringer stiffened shell, with $2a = 12$ in., $W_{sf}/W_0 = 0.5$, and a frame at position 6. The plots of linear prebuckling deflections and stresses are nearly identical to those for a shell without the reinforcing frame. The normalized buckling displacements are shown in Fig. 11. The

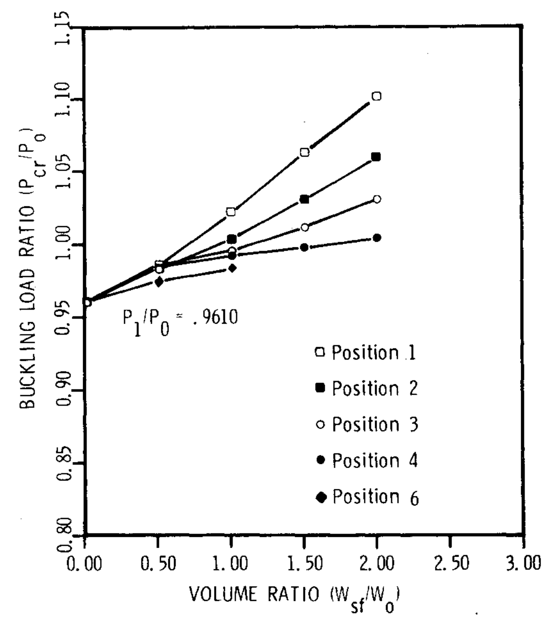

Fig. 12 P_{cr}/P_0 **vs** W_{sf}/W_0 **for stringer stiffened shells with cutout size** $2a = 12$ **in.**

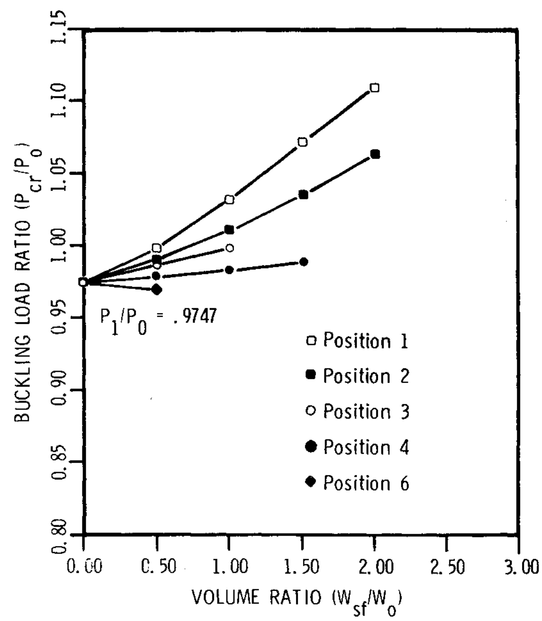

Fig. 14 P_{cr}/P_0 **vs** W_{sf}/W_0 **for ring and stringer stiffened shells with cutout size** $2a = 12$ **in.**

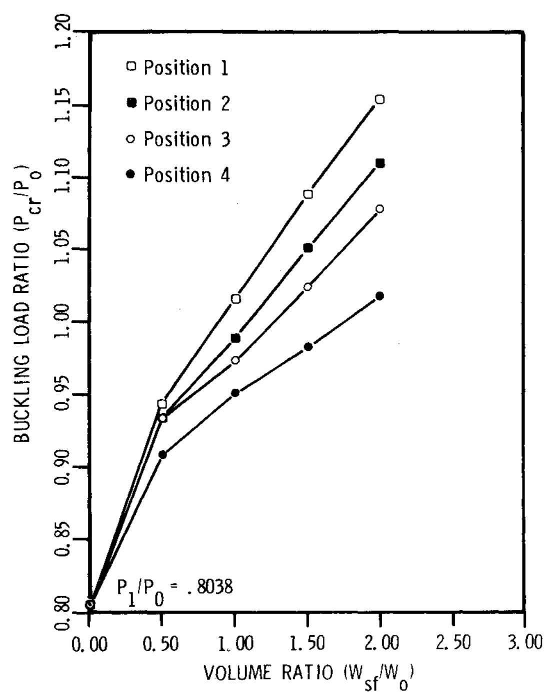

Fig. 13 P_{cr}/P_0 **vs** W_{sf}/W_0 **for stringer stiffened shells with cutout size** $2a = 24$ **in.**

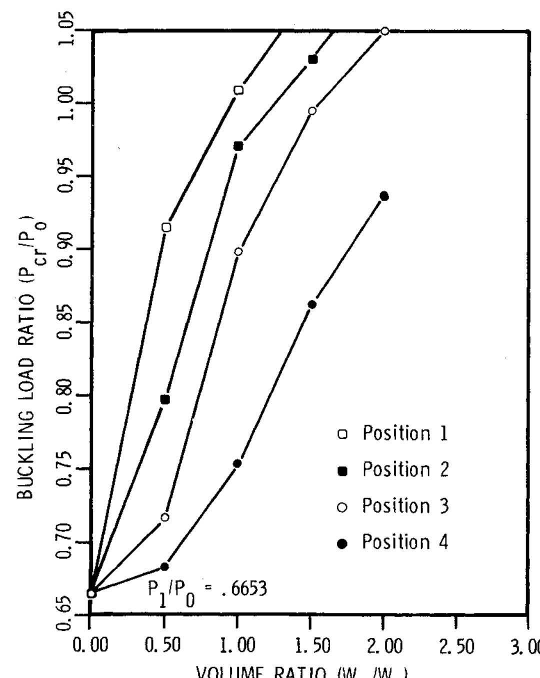

Fig. 15 P_{cr}/P_0 **vs** W_{sf}/W_0 **for ring and stringer stiffened shells with cutout size** $2a = 24$ **in.**

buckling pattern is similar to that of the shell without a frame (not shown). Also, notice that the vertical leg of the frame lies at the bottom of an inward buckle. It is thus supposed that, for a given frame position, the eccentricity effect of the internal frame may contribute to the early buckling of the shell due to the additional compressive stress caused by the frame's eccentricity leading to a less stable configuration.

In Figs. 12-15, buckling load ratio P_{cr}/P_0 is plotted vs volume ratio W_{sf}/W_0. The variation of shell stiffening and cutout size is in the same order as for Figs. 2-5. As a general rule, the heavier reinforcing frames lead to higher buckling ratios (P_{cr}/P_0). An exception to this trend is shown in Fig. 14 for frame position 6. Notice that for stringer stiffened shells, (Figs. 12 and 13) the light frames ($W_{sf}/W_0 = 0.5$) produce a strenghtening effect that is roughly independent of frame position. The effect of frame position does not become large until heavier frames are used. On the other hand, for ring and stringer stiffened shells (Figs. 14 and 15) the effect of frame position is very large for light frames ($W_{sf}/W_0 = 0.5$).

Apparently, the light frames for ring and stringer stiffened shells are affecting a factor of buckling which is not significant in stringer stiffened shells. An interesting observation is that the ring and stringer stiffened shells already have a series of reinforcing frames surrounding the cutout. The additional stiffness of this configuration requires heavier frames in order to alter the deformations significantly, whereas the relatively flexible stringer stiffened shell does not require heavy frames to alter the buckling pattern. In order to give a more detailed explanation of this effect, consider the stiffening effect of the ring of the reinforcing frame. Since the volume of reinforcing frame remains constant for a given volume ratio, W_{sf}/W_0, the cross-sectional area of the frame is inversely proportional to the distance d of the frame from the centerline of the cutout and the moment of inertia to the square of d. Hence, for the ring and stringer stiffened shell, the added stiffness of the reinforcing frame is large when the frame is along the cutout edge; but for the frame removed from the cutout it is not as large when compared to the stiffness of the existing ring stiffened skin. On the other hand, for the stringer stiffened shell, the increase in stiffness in the circumferential direction is large even for a frame removed from the cutout; since without the frame, only the relatively weak skin resists bending and compressive loads.

Note that Figs. 12-15 can be used to determine the required combination of frame volume and frame location required to achieve a certain ratio P_{cr}/P_0. For example, consider Fig. 15, which is for a ring and stringer stiffened shell with a 24×24 in. cutout; suppose that it is not possible to reinforce the cutout at the edge, but reinforcement can be placed at position 2, what is the required volume of reinforcing frame to achieve a buckling load equivalent to a similar shell without a cutout? The required volume ratio of the reinforcing frame can be easily determined as follows: Draw a line through $P_{cr}/P_0 = 1.0$ that intersects the frame position 2 curve. A line dropped from this intersection to the abcissa yields the required volume ratio (and therefore volume) for the reinforcing frame. Note that this procedure works equally well for any other value of P_{cr}/P_0.

Conclusions

The following conclusions can be made.

1) A reinforcing frame equal in volume to the material removed for a cutout and placed adjacent to that cutout, can increase the buckling load of the shell to or above that of a shell without a cutout.

2) For either a stringer or a ring and stringer stiffened cylindrical shell with rectangular cutouts, the optimum position for a reinforcing frame is along the cutout edges. Yet, the designer must be aware that in certain situations a frame positioned far removed from the cutout edge may reduce the buckling load below that of an unreinforced shell.

3) Positioning a frame away from the cutout edge, in most cases, drastically reduces the buckling load from that obtainable by placing the same volume of reinforcing frame adjacent to the cutout.

4) For very light frames there may be a frame position away from the cutout edge which is equally effective as the position adjacent to the cutout.

5) It is possible, with the use of STAGS to determine the reinforcement frame cross section necessary to efficiently stiffen a shell with cutouts to any desired strength.

References

[1]Starnes, J.H., Jr., "The Effect of a Circular Hole on the Buckling of Cylindrical Shells," Ph.D Thesis, California Institute of Technology, Pasadena, Calif., 1970.

[2]Singer, J., "Buckling of Integrally Stiffened Cylindrical Shells—a Review of Experiment and Theory," *Contributions to the Theory of Aircraft Structures,* Delft University Press, 1972, pp. 325-358.

[3]Palazotto, A.N., "Bifurcation and Collapse Analysis of Stringer and Ring Stringer Stiffened Cylindrical Shells with Cutouts," *Computers and Structures,* Vol. 7, Feb. 1977, pp. 47-58.

[4]Almroth, B.O., Brogan, F.A., and Zele, F., "User's Manual for the STAGS-A Computer Code," *Collapse Analysis for Shells of General Shape,* Vol. II, Air Force Flight Dynamics Laboratory, AFFDL-TR-71-8, March 1973.

[5]Timoshenko, S.P. and Goodier, J.N., *Theory of Elasticity* (3rd ed), McGraw-Hill Book Company, New York, 1970.